Due August 5, 11:59pm on Gradescope.

The following are warm-up exercises and are *not* to be turned in. You may treat these as extra practice problems.

10.2.23, 10.2.26, 10.2.54, 10.3.32, 10.3.44, 10.3.50, 10.3.71, 10.4.21, 10.4.28, 10.4.36, 10.4.43.

Turn in the following exercises. Remember to carefully justify every statement that you write, and to follow the style of proper mathematical writing. You may cite any result proved in the textbook or lecture, unless otherwise mentioned. Each problem is worth 10 points with parts weighted equally, unless otherwise mentioned.

1. 10.2.32.

2. Let S be a finite set with $n \geq 1$ elements, and let a, b be positive integers with $ab = n$. We define an *a-partition* of S to be a collection of a equally sized disjoint subsets $U_1, \ldots, U_a$ of S, whose union is S (so the U_i all have size b). Note that this is just a special type of set partition where all the subsets have the same size. We also say a subset of a elements $\{s_1, \ldots, s_a\}$ of S is a *set of representatives* for a partition $\{U_1, \ldots, U_a\}$ if each U_i contains exactly one s_j (possibly $i \neq j$). Now, suppose we are given two a-partitions $\{U_1, \ldots, U_a\}$ and $\{V_1, \ldots, V_a\}$ of S. Show that there is a subset $\{s_1, \ldots, s_a\} \subseteq S$ which is simultaneously a set of representatives for both partitions.

   Remark: The above result was Hall's original group-theoretic motivation for the "marriage theorem." In that context, S is a finite group, and the two partitions of S in question are the set of *left cosets* and the set of *right cosets* of a subgroup of S. Hence the above result gives a common set of representatives for the left and right cosets.

3. 10.2.66.

4. 10.3.46.

5. 10.3.56. See 10.2.61 for the definition of the complementary graph $\overline{G}$.

6. 10.4.22.

7. 10.4.44. You may assume the result of 10.4.43, although it is a good exercise to prove it.

8. **(Bonus Problem, 5 points)** For your hard work this summer, you get 5 bonus points for free. You can use this space to mention problems from the homework or topics/theorems from the course you liked in particular, or ones that you didn't (other feedback is welcome as well). You also don't have to write anything at all, and you'll still get the bonus points.